Pergamon

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A CONSTITUTIVE THEORY FOR POROUS COMPOSITE MATERIALS

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Abstract—A phenomenological model for porous layered materials has been developed. The phenomenological approach for layered materials is combined with a poroelastic constitutive model. Explicit expressions for effective elastic moduli, thermal expansion coefficients, and poroelastic moduli are obtained. The obtained results reduce to those of layered materials when there are no pores. The obtained model is applied to failure analysis of thermochemically decomposing composites subjected to high temperature and high heating rates. A separate analysis of carbon fiber and phenolic resin responses explains why carbon fiber shrinkage causes more delamination failure. © 1998 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Layered materials represent one of the most widely used forms of composites today. There are some cases where the layered materials are porous. For example, carbon fiber reinforced plastic composites used as heat shields or heat liners thermochemically decompose and become porous. Gas generated due to the decomposition is often trapped inside the pores and exerts a high pore pressure. Apart from the man-made composites, naturally occurring geological materials can be idealized as layered porous materials saturated with gas, water and oil. Therefore, it is important to know the effect of microstructure and material properties of constituents on the overall response of porous layered materials.

Micromechanical constitutive models have been developed for layered materials. Postma (1955) obtained the five effective elastic moduli of transversely isotropic layered materials in terms of the four elastic moduli of isotropic constituent layers (two elastic moduli for each layer) and the thickness ratio. Backus (1962) considered the inverse problem and derived the conditions on the five elastic coefficients of a homogeneous transversely isotropic medium which are necessary and sufficient for the medium to be equivalent to a horizontally layered isotropic medium. Salamon (1968) obtained the results similar to Postma (1955) for transversely isotropic layers. Chou, Carleone and Hsu (1972) further generalized the results for anisotropic layers. Hoffman (1973) considered the thermal expansion coefficient mismatch problems. The mechanical response beyond elastic range was discussed in Aboudi (1991). In all of these models, the constituent layers are assumed to be non-porous solids. When at least one of the constituent layers is porous, the pore pressure greatly influences the overall material response of the composite, and problems cannot be analyzed based on the existing micro mechanical models for layered materials.

Micromechanical models for a fluid-filled porous material have been well developed. The notion of "effective stress" is often used to describe the influence of pore fluid pressure on various mechanical response properties of fluid-filled porous materials. Explicit expressions of an effective stress for an isotropic porous material were obtained by Biot and Willis (1957), and later by Nur and Byerlee (1971), and those for an anisotropic porous material were obtained by Carroll (1979), Carroll and Katsube (1983), and Katsube (1988).

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In this work, a phenomenological constitutive model for layered porous materials is developed. The established poroelastic constitutive theory is incorporated into the phenomenological model for layered materials. The effective elastic moduli, thermal expansion coefficients, and poroelastic constants are expressed in terms of the constituent material properties and the thickness ratio. Explicit expressions of the internal stress components in each constituent layer are also obtained. The results reduce to those of the existing models for layered materials when there are no pores. The developed model is applied to thermochemically decomposing carbon–phenolic composites subjected to high temperature, high heating rates and overall loading. The model explains why carbon fiber shrinkage in the fiber direction causes more delamination failures.

2. THERMOMECHANICAL RESPONSE OF COMPOSITES

2.1. *Basic assumptions*

A representative volume element of a porous layered material will be considered. The micro-structure of the composite consists of alternating layers of two different types of porous materials with in-plane x_1 and x_2 axes and out of plane x_3 axis. Both layers and pore structures are assumed to be transversely isotropic with the plane of isotropy parallel to the layers. The ratio of the thickness of two layers is determined by the volumetric ratio. Let $\eta^{(1)}$ and $\eta^{(2)}$ represent the volumetric fraction of layer 1 and 2, respectively, so that

$$\eta^{(1)}+\eta^{(2)} = 1 \tag{1}$$

The volume average response of the composite will be considered. For this purpose, the volume average stress and strain components are introduced. The overall stress σ_{ij} is expressed as the weighted sum of layer 1 stress $\sigma_{ij}^{(1)}$ and layer 2 stress $\sigma_{ij}^{(2)}$ as in eqn (2). Similarly, the overall strain ε_{ij} is expressed as the weighted sum of layer 1 strain $\varepsilon_{ij}^{(1)}$ and layer 2 strain $\varepsilon_{ij}^{(2)}$ as in eqn (3).

$$\sigma_{ij} = \eta^{(1)}\sigma_{ij}^{(1)}+\eta^{(2)}\sigma_{ij}^{(2)} \tag{2}$$

$$\varepsilon_{ij} = \eta^{(1)}\varepsilon_{ij}^{(1)}+\eta^{(2)}\varepsilon_{ij}^{(2)} \tag{3}$$

Assuming that each layer is porous with porosity $\Phi^{(\gamma)}$, the layer stress $\sigma_{ij}^{(\gamma)}$ consists of solid stress $\sigma_{ij}^{s(\gamma)}$ and pore pressure P_g as in eqn (4). Similarly, the layer strain $\varepsilon_{ij}^{(\gamma)}$ consists of solid strain $\varepsilon_{ij}^{s(\gamma)}$ and pore space strain $\varepsilon_{ij}^{p(\gamma)}$ as in eqn (5). The definitions of solid stress $\sigma_{ij}^{s(\gamma)}$, pore pressure P_g, solid strain $\varepsilon_{ij}^{s(\gamma)}$, and pore space strain $\varepsilon_{ij}^{p(\gamma)}$ are identical to those by Katsube (1985).

$$\sigma_{ij}^{(\gamma)} =(1-\Phi^{(\gamma)})\sigma_{ij}^{s(\gamma)}-\Phi^{(\gamma)}P_g\delta_{ij}, \quad \gamma = 1,2 \tag{4}$$

$$\varepsilon_{ij}^{(\gamma)} =(1-\Phi^{(\gamma)})\varepsilon_{ij}^{s(\gamma)}+\Phi^{(\gamma)}\varepsilon_{ij}^{p(\gamma)}, \quad \gamma = 1,2 \tag{5}$$

Assuming that the individual layers are perfectly bonded together, the volume average in-plane strain components of layer 1 and those of layer 2 can be assumed to be the same. Since the surface forces at the layer interface are continuous, the volume average out-of-plane stress components of layer 1 and those of layer 2 can be assumed to be the same. These two assumptions lead to eqn (6) and (7).

$$\varepsilon_{\alpha\beta} = \varepsilon_{\alpha\beta}^{(1)} = \varepsilon_{\alpha\beta}^{(2)} \tag{6}$$

$$\sigma_{i3} = \sigma_{i3}^{(1)} = \sigma_{i3}^{(2)} \tag{7}$$

where α and β are 1 or 2, and i takes an integer from 1–3.

2.2. *Poroelastic assumption*

Applying the thermomechanical response of a poroelastic material (see Carroll, 1979) to each layer, we have

$$\varepsilon_{ij}^{(\gamma)} = C_{ijkl}^{(\gamma)}\hat{\sigma}_{kl}^{(\gamma)} + \beta_{ij}^{(\gamma)}T \tag{8}$$

where the effective stress tensor is defined by

$$\hat{\sigma}_{ij}^{(\gamma)} = \sigma_{ij}^{(\gamma)} + P_g\alpha_{ij}^{(\gamma)} \tag{9}$$

and

$$\alpha_{ij}^{(\gamma)} = \delta_{ij} - M_{ijkl}^{(\gamma)}C_{klmm}^{s(\gamma)} \tag{10}$$

The volumetric strain for pore space (see Katsube, 1988) is given by

$$\varepsilon_{mm}^{p(\gamma)} = C_{mmkl}^{p(\gamma)}\{\sigma_{kl}^{(\gamma)}\} + \beta_{mm}^{s(\gamma)}T \tag{11}$$

where

$$C_{ijkl}^{p(\gamma)} = \frac{1}{\Phi^{(\gamma)}}(C_{ijkl}^{(\gamma)} - C_{ijkl}^{s(\gamma)}) \tag{12}$$

$$\{\sigma_{ij}^{(\gamma)}\} = \sigma_{ij}^{(\gamma)} + P_g(\delta_{ij} - M_{ijkl}^{p(\gamma)}C_{klmm}^{s(\gamma)}) \tag{13}$$

In the above equations, $M_{ijkl}^{(\gamma)}$ and $\beta_{ij}^{(\gamma)}$ are the components of the elastic moduli tensor and those of the thermal expansion tensor for a dry porous material of layer γ, respectively. Temperature increase, the fluid pore pressure, the components of overall layer γ stress tensor, those of the poroelastic tensor for layer γ, the elastic compliance tensor components for the solid material of layer γ, and the components of the thermal expansion tensor for the solid material of layer γ are, respectively, denoted by $T, P_g, \sigma_{ij}^{(\gamma)}, \alpha_{ij}^{(\gamma)}, C_{ijkl}^{s(\gamma)}$ and $\beta_{ij}^{s(\gamma)}$.

For a transversely isotropic composite, the above equations reduce to

$$\varepsilon_{11}^{(\gamma)} = \frac{1}{E_T^{(\gamma)}}\sigma_{11}^{(\gamma)} - \frac{\nu_T^{(\gamma)}}{E_T^{(\gamma)}}\sigma_{22}^{(\gamma)} - \frac{\nu_{LT}^{(\gamma)}}{E_L^{(\gamma)}}\sigma_{33}^{(\gamma)} + \frac{(1-\nu_T^{(\gamma)})\alpha_T^{(\gamma)} - \nu_{TL}^{(\gamma)}\alpha_L^{(\gamma)}}{E_T^{(\gamma)}}P_g + \beta_T^{(\gamma)}T \tag{14a}$$

$$\varepsilon_{22}^{(\gamma)} = -\frac{\nu_T^{(\gamma)}}{E_T^{(\gamma)}}\sigma_{11}^{(\gamma)} + \frac{1}{E_T^{(\gamma)}}\sigma_{22}^{(\gamma)} - \frac{\nu_{LT}^{(\gamma)}}{E_L^{(\gamma)}}\sigma_{33}^{(\gamma)} + \frac{(1-\nu_T^{(\gamma)})\alpha_T^{(\gamma)} - \nu_{TL}^{(\gamma)}\alpha_L^{(\gamma)}}{E_T^{(\gamma)}}P_g + \beta_T^{(\gamma)}T \tag{14b}$$

$$\varepsilon_{33}^{(\gamma)} = -\frac{\nu_{LT}^{(\gamma)}}{E_L^{(\gamma)}}\sigma_{11}^{(\gamma)} - \frac{\nu_{LT}^{(\gamma)}}{E_L^{(\gamma)}}\sigma_{22}^{(\gamma)} + \frac{1}{E_L^{(\gamma)}}\sigma_{33}^{(\gamma)} + \frac{\alpha_L^{(\gamma)} - 2\nu_{LT}^{(\gamma)}\alpha_T^{(\gamma)}}{E_L^{(\gamma)}}P_g + \beta_L^{(\gamma)}T \tag{14c}$$

$$\varepsilon_{12}^{(\gamma)} = \frac{1}{2\mu_T^{(\gamma)}}\sigma_{12}^{(\gamma)} \tag{14d}$$

$$\varepsilon_{23}^{(\gamma)} = \frac{1}{2\mu_{LT}^{(\gamma)}}\sigma_{23}^{(\gamma)} \tag{14e}$$

$$\varepsilon_{31}^{(\gamma)} = \frac{1}{2\mu_{LT}^{(\gamma)}}\sigma_{31}^{(\gamma)} \tag{14f}$$

$$
\begin{aligned}
\varepsilon_{mm}^{P(\gamma)} = {} & \frac{(1-\nu_T^{(\gamma)})\alpha_T^{(\gamma)} - \nu_{TL}^{(\gamma)}\alpha_L^{(\gamma)}}{\Phi^{(\gamma)}E_T^{(\gamma)}}(\sigma_{11}^{(\gamma)}+\sigma_{22}^{(\gamma)}) + \frac{\alpha_L^{(\gamma)} - 2\nu_{LT}^{(\gamma)}\sigma_T^{(\gamma)}}{\Phi^{(\gamma)}E_L^{(\gamma)}}\sigma_{33}^{(\gamma)} \\
& + \frac{P_g(1+\Phi^{(\gamma)})}{\Phi^{(\gamma)}}\left[\frac{2\{(1-\nu_T^{(\gamma)})\alpha_T^{(\gamma)} - \nu_{TL}^{(\gamma)}\,\alpha_L^{(\gamma)}\}}{E_L^{(\gamma)}} + \frac{\alpha_L^{(\gamma)} - 2\nu_{LT}^{(\gamma)}\alpha_T^{(\gamma)}}{E_L^{(\gamma)}}\right] \\
& - P_g\left[\frac{2(1-\nu_T^{(\gamma)}-\nu_{TL}^{(\gamma)})}{E_T^{(\gamma)}} - \frac{1-2\nu_{LT}^{(\gamma)}}{E_L^{(\gamma)}}\right] + (2\beta_T^{s(\gamma)} + \beta_L^{s(\gamma)})T
\end{aligned}
\tag{15}
$$

The thermal expansion coefficients in the in-plane and across-ply directions of layer γ and those of the solid material in layer γ are denoted by $\beta_T^{(\gamma)}$ and $\beta_L^{s(\gamma)}$, $\beta_T^{s(\gamma)}$ and $\beta_L^{s(\gamma)}$, respectively. The corresponding poroelastic constants are represented by $\alpha_T^{(\gamma)}$ and $\alpha_L^{(\gamma)}$. $E_T^{(\gamma)}$ and $E_L^{(\gamma)}$, respectively, are the in-plane and across-ply Young's moduli. $\nu_{TL}^{(\gamma)}$, $\nu_{LT}^{(\gamma)}$ and $\nu_T^{(\gamma)}$ are the Poisson's ratios. The in-plane and across-ply shear moduli are denoted by $\mu_T^{(\gamma)}$ and $\mu_{LT}^{(\gamma)}$. Among the seven elastic constants, only five are independent.

The same form of response equations can be used for a porous layered composite as a whole. Equations (8)–(13) and eqns (14) can be written without superscript (γ).

2.3. *Effective material constants*

For layer 1, layer 2, and the composite, there are six stress components each and six strain components each. Therefore there are 36 variables. There are six response equations each for two layers as in eqns (8). The number of independent equations from eqns (2), (3), (6), (7) is 18. Therefore, the total number of independent equations so far is 30. After a lengthy algebra, the six stress components of the composite can be expressed in terms of the six strain components of the composite. The obtained equations are in the form of eqns (14) with coefficients expressed in terms of the constituent layer material constants. Comparing the corresponding coefficients in these two sets of equations, the effective material constants are obtained as follows.

(i) Effective Thermal Expansion

$$
\beta_T = \frac{\eta^{(2)}(1-\nu_T^{(1)})E_T^{(2)}\beta_T^{(2)} + \eta^{(1)}(1-\nu_T^{(2)})E_T^{(1)}\beta_T^{(1)}}{L}
\tag{16a}
$$

$$
\beta_L = \eta^{(2)}\beta_L^{(2)} + \eta^{(1)}\beta_L^{(1)} + \frac{2\eta^{(2)}\eta^{(1)}}{L}(\nu_{TL}^{(2)}E_T^{(1)} - \nu_{TL}^{(1)}E_T^{(2)})(\beta_T^{(2)} - \beta_T^{(1)})
\tag{16b}
$$

(ii) Effective Poroelastic Parameters

$$
\begin{aligned}
\frac{1-\nu_T}{E_T}\alpha_T - \frac{\nu_{TL}}{E_T}\alpha_L = {} & \eta^{(2)}(1-\nu_T^{(1)})\frac{(1-\nu_T^{(2)})\alpha_T^{(2)} - \nu_{TL}^{(2)}\alpha_L^{(2)}}{L} \\
& + \eta^{(1)}(1-\nu_T^{(2)})\frac{(1-\nu_T^{(1)})\alpha_T^{(1)} - \nu_{TL}^{(1)}\alpha_L^{(1)}}{L}
\end{aligned}
\tag{17a}
$$

$$
\begin{aligned}
-\frac{2\nu_{TL}}{E_T}\alpha_T + \frac{1}{E_L}\alpha_L = {} & \frac{2\eta^{(1)}\eta^{(2)}}{L}[(1-\nu_T^{(2)})\alpha_T^{(2)} - \nu_{TL}^{(2)}\alpha_L^{(2)}]\left[\frac{\nu_{TL}^{(2)}E_T^{(1)} - \nu_{TL}^{(1)}E_T^{(2)}}{E_T^{(2)}}\right] \\
& - \frac{2\eta^{(1)}\eta^{(2)}}{L}[(1-\nu_T^{(1)})\alpha_T^{(1)} - \nu_{TL}^{(1)}\alpha_L^{(1)}]\left[\frac{-\nu_{TL}^{(1)}E_T^{(2)} + \nu_{TL}^{(2)}E_T^{(1)}}{E_T^{(1)}}\right] \\
& + \eta^{(2)}\frac{\alpha_L^{(2)} - 2\nu_{LT}^{(2)}\alpha_T^{(2)}}{E_L^{(2)}} + \eta^{(1)}\frac{\alpha_L^{(1)} - 2\nu_{LT}^{(1)}\alpha_T^{(1)}}{E_L^{(1)}}
\end{aligned}
\tag{17b}
$$

(iii) Effective Elastic Constants

$$E_L = \frac{L}{\left(\frac{\eta^{(2)}}{E_L^{(2)}} + \frac{\eta^{(1)}}{E_L^{(1)}}\right) L - 2\eta^{(2)}\eta^{(1)}(v_{TL}^{(2)}E_T^{(1)} - v_{TL}^{(1)}E_T^{(2)}) \left(\frac{v_{TL}^{(2)}}{E_T^{(2)}} - \frac{v_{TL}^{(1)}}{E_T^{(1)}}\right)} \tag{18a}$$

$$v_{LT} = \frac{\eta^{(2)}(1 - v_T^{(1)})v_{TL}^{(2)} + \eta^{(1)}(1 - v_T^{(2)})v_{TL}^{(1)}}{\left(\frac{\eta^{(2)}}{E_L^{(2)}} + \frac{\eta^{(1)}}{E_L^{(1)}}\right) L - 2\eta^{(2)}\eta^{(1)}(v_{TL}^{(2)}E_T^{(1)} - v_{TL}^{(1)}E_T^{(2)}) \left(\frac{v_{TL}^{(2)}}{E_T^{(2)}} - \frac{v_{TL}^{(1)}}{E_T^{(1)}}\right)} \tag{18b}$$

$$E_T = \frac{LH}{\eta^{(2)}[1 - (v_T^{(1)})^2]E_T^{(2)} + \eta^{(1)}[1 - (v_T^{(2)})^2]E_T^{(1)}} \tag{18c}$$

$$v_T = \frac{\eta^{(2)}[1 - (v_T^{(1)})^2]v_T^{(2)}E_T^{(2)} + \eta^{(1)}[1 - (v_T^{(2)})^2]v_T^{(1)}E_T^{(1)}}{\eta^{(2)}[1 - (v_T^{(1)})^2]E_T^{(2)} + \eta^{(1)}[1 - (v_T^{(2)})^2]E_T^{(1)}} \tag{18d}$$

$$v_{TL} = \frac{\eta^{(2)}(1 - v_T^{(1)})v_{TL}^{(2)} + \eta^{(1)}(1 - v_T^{(2)})v_{TL}^{(1)}}{\eta^{(2)}[1 - (v_T^{(1)})^2]E_T^{(2)} + \eta^{(1)}[1 - (v_T^{(2)})^2]E_T^{(1)}} H \tag{18e}$$

$$\mu_{LT} = \frac{\mu_{LT}^{(1)}\mu_{LT}^{(2)}}{\eta^{(1)}\mu_{LT}^{(2)} + \eta^{(2)}\mu_{LT}^{(1)}} \tag{18f}$$

$$\mu_T = \eta^{(1)}\mu_T^{(1)} + \eta^{(2)}\mu_T^{(2)} \tag{18g}$$

where

$$L = \eta^{(1)}(1 - v_T^{(2)})E_T^{(1)} + \eta^{(2)}(1 - v_T^{(1)})E_T^{(2)} \tag{19a}$$

$$H = \eta^{(1)}(1 + v_T^{(2)})E_T^{(1)} + \eta^{(2)}(1 + v_T^{(1)})E_T^{(2)} \tag{19b}$$

It can be shown that eqns (18) are identical to those obtained by Salamon (1968). Equations (17) give explicit expressions of effective poroelastic parameters in terms of material properties of constituent layers. If we let the material properties of each constituent layer be the same in eqns (17), the effective poroelastic parameters can be shown to be identical to the poroelastic parameters of the constituent layers as follows:

$$\alpha_T = \alpha_T^{(1)} = \alpha_T^{(2)} \tag{20a}$$

$$\alpha_L = \alpha_L^{(1)} = \alpha_L^{(2)} \tag{20b}$$

2.4. *Internal stress components*

The 30 equations discussed above can also be used to express the stress components of the constituents in terms of those of the composite. The results are given by eqns (21) for layer 1. The results for layer 2 can be obtained by interchanging the superscript 1 and 2 in eqns (21).

$$\begin{aligned}\sigma_{11}^{(1)} = {} & \frac{\eta^{(2)}E_T^{(1)}E_T^{(2)}}{L}(\beta_T^{(2)} - \beta_T^{(1)})T \\ & + \frac{\eta^{(2)}[(1 - v_T^{(2)})\alpha_T^{(2)} - v_{TL}^{(2)}\alpha_L^{(2)}]E_T^{(1)} - \eta^{(2)}[(1 - v_T^{(1)})\alpha_T^{(1)} - v_{TL}^{(1)}\alpha_L^{(1)}]E_T^{(2)}}{L} P_g \\ & + \frac{E_T^{(1)}\{\eta^{(2)}(1 - v_T^{(1)}v_T^{(2)})E_T^{(2)} + \eta^{(1)}[1 - (v_T^{(2)})^2]E_T^{(1)}\}}{LH}\sigma_{11} \\ & - \frac{E_T^{(2)}E_T^{(1)}\eta^{(2)}(v_T^{(2)} - v_T^{(1)})}{LH}\sigma_{22} + \frac{\eta^{(2)}(v_{TL}^{(1)}E_T^{(2)} - v_{TL}^{(2)}E_T^{(1)})}{L}\sigma_{33}\end{aligned} \tag{21a}$$

$$
\begin{aligned}
\sigma_{22}^{(1)} = {} & \frac{\eta^{(2)} E_T^{(1)} E_T^{(2)}}{L} (\beta_T^{(2)} - \beta_T^{(1)}) T \\
& + \frac{\eta^{(2)} [(1 - \nu_T^{(2)}) \alpha_T^{(2)} - \nu_{TL}^{(2)} \alpha_L^{T}] E_T^{(1)} - \eta^{(2)} [(1 - \nu_T^{(1)}) \alpha_T^{(1)} - \nu_{TL}^{(1)} \alpha_L^{(1)}] E_T^{(2)}}{L} P_g \\
& + \frac{E_T^{(1)} E_T^{(2)} \eta^{(2)} (\nu_T^{(1)} - \nu_T^{(2)})}{LH} \sigma_{11} \\
& + \frac{E_T^{(1)} \{\eta^{(2)} (1 - \nu_T^{(1)} \nu_T^{(2)}) E_T^{(2)} + \eta^{(1)} [1 - (\nu_T^{(2)})^2] E_T^{(1)}\}}{LH} \sigma_{22} \\
& + \frac{\eta^{(2)} (\nu_{TL}^{(1)} E_T^{(2)} - \nu_{TL}^{(2)} E_T^{(1)})}{L} \sigma_{33} \qquad \text{(21b)}
\end{aligned}
$$

$$\sigma_{33}^{(1)} = \sigma_{33} \qquad \text{(21c)}$$

$$\sigma_{12}^{(1)} = \frac{\mu_T^{(1)}}{\eta^{(1)} \mu_T^{(1)} + \eta^{(2)} \mu_T^{(2)}} \sigma_{12} \qquad \text{(21d)}$$

$$\sigma_{13}^{(1)} = \sigma_{13} \qquad \text{(21e)}$$

$$\sigma_{23}^{(1)} = \sigma_{23} \qquad \text{(21f)}$$

Hoffman (1973) obtained the explicit expressions for the internal stress components of nonporous layered materials when the composite is subjected to temperature increase. Ignoring the pore pressure and overall surface traction terms, and using isotropic constituent layers with identical Poisson's ratio, eqns (21) reduce to those obtained by Hoffman (1973).

3. APPLICATIONS

3.1. *Basic assumptions*

The developed model will be applied to failure analysis of thermochemically decomposing carbon–phenolic composite materials. Carbon–phenolic composite materials are used as heat liners for the solid rocket motor nozzles of space shuttles. The composite is made of layers of woven carbon fiber impregnated in phenolic resin. When the composite is exposed to an extreme thermo-chemical environment, it decomposes and forms a char. The gas generated by decomposition is often trapped inside the composite and exerts high pore pressures. It can cause separation of layers and lead to premature end of service life as a heat liner. This delamination failure is known as ply lift and has been investigated by many researchers.

Finite element computer codes have been developed for carbon–phenolic composite by Sullivan and Salamon (1992), McManus and Springer (1992), Kuhlmann (1991) and Wu and Katsube (1997). In these codes, a porous composite is modeled as a whole and therefore, the material properties of carbon fiber and resin cannot be handled separately. When carbon fiber which undergoes thermal shrinkage is used in composites, the ply lift failures are observed more often than before. The material properties (other than thermal shrinkage) of the carbon fiber with thermal shrinkage are the same as those of the carbon fiber without thermal shrinkage. In order to investigate the effect of fiber shrinkage on the overall material behavior of the composite, we will use the phenomenological porous composite theory developed in the previous section.

In reality, there is no clear distinction between woven carbon layer and resin layer since some resin is inside the woven carbon. However, there is resin between each layer of woven carbon. When the composite is subjected to thermochemical environment, only resin decomposes and becomes porous. In this work, we idealize a carbon–phenolic material as a layered composite with a porous resin layer and a carbon layer without pores. Using superscript (1) for a resin layer and superscript (2) for a carbon layer, the porosity of carbon layer and poroelastic constants for carbon layer are zero as in eqn (22).

$$\Phi^{(2)} = \alpha_T^{(2)} = \alpha_L^{(2)} = 0 \tag{22}$$

The resin layer consists of the solid resin and the pore space, and the solid resin is assumed to be isotropic. With this assumption, eqn (10) can be simplified, and the coefficients of P_g in eqns (14a) and (14b) can be rewritten as follows:

$$\frac{(1-v_T^{(1)})\alpha_T^{(1)} - v_{TL}^{(1)}\alpha_L^{(1)}}{E_T^{(1)}} = \frac{1-v_T^{(1)}-v_{TL}^{(1)}}{E_T^{(1)}} - \frac{1}{3K^{s(1)}} \tag{23a}$$

$$\frac{\alpha_L^{(1)} - 2v_{LT}^{(1)}\alpha_T^{(1)}}{E_L^{(1)}} = \frac{1-2v_{LT}^{(1)}}{E_L^{(1)}} - \frac{1}{3K^{s(1)}} \tag{23b}$$

where $K^{s(1)}$ is the bulk modulus of solid resin. Similarly, using eqns (23a) and (23b), eqn (15) can be rewritten as follows:

$$\begin{aligned}\varepsilon_{mm}^{(1)P} &= (2\beta_T^{s(1)} + \beta_L^{s(1)})T + \frac{1}{\Phi^{(1)}}\left(\frac{1-v_T^{(1)}-v_{TL}^{(1)}}{E_T^{(1)}} - \frac{1}{3K^{s(1)}}\right)(\sigma_{11}^{(1)} + \sigma_{22}^{(1)}) \\ &\quad + \frac{1}{\Phi^{(1)}}\left(\frac{1-2v_{LT}^{(1)}}{E_L^{(1)}} - \frac{1}{3K^{s(1)}}\right)\sigma_{33}^{(1)} + \frac{1}{\Phi^{(1)}}\left[\frac{2(1-v_T^{(1)}-v_{TL}^{(1)})}{E_T^{(1)}} + \frac{1-2v_{LT}^{(1)}}{E_L^{(1)}} - \frac{1+\Phi^{(1)}}{K^{s(1)}}\right]P_g\end{aligned} \tag{24}$$

We assume that the pore gas generated by decomposition satisfies the perfect gas state equation, and that the generated gas is trapped inside the composite. The validity of these assumptions are shown in Wu and Katsube (1997) where the numerical predictions based on the same assumptions are verified against experimental data. As in Wu and Katsube (1996), under the infinitesimal deformation assumption, the gas pressure inside the pores can be calculated from eqn (25).

$$P_g = \frac{m_g R T_a}{M V_{pl}}(1 - \varepsilon_{mm}^{(1)P}) \tag{25}$$

where m_g, R, M, T_a and V_{pl}, respectively, are the gas mass trapped inside the pores, universal gas constant, molecular weight, absolute temperature, and the pore volume after solid mass loss.

3.2. *Free thermal expansion tests*

The delamination failure discussed in the previous section is normally observed near the exit of the nozzles. Therefore, the composite can be assumed to be more or less free from the overall mechanical stress and subjected to high pore pressure inside the composite. Experiments where composite samples are uniformly heated without any external outer boundary are known as free thermal expansion tests. It has been shown that the free thermal expansion tests cause delamination failures if gas escape from the sample is prevented. In this section, we will simulate the volume average response of the free thermal expansion tests and investigate why carbon fiber shrinkage cause more delamination failures.

Using the overall stress free condition on the outer boundary of the composite and eqn (7), we have eqn (26).

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{33}^{(1)} = \sigma_{33}^{(2)} = 0 \tag{26}$$

The loading condition and eqns (21a) and (21b) lead to eqn (27).

$$\sigma_{11}^{(\gamma)} = \sigma_{22}^{(\gamma)}, \quad \gamma = 1,2 \tag{27}$$

Using eqn (26) and eqn (23a), eqn (21a) reduces to eqn (28).

$$\sigma_{11}^{(1)} = \frac{\eta^{(2)}E_T^{(1)}E_T^{(2)}}{L}(\beta_T^{(2)}-\beta_T^{(1)})T - \frac{\eta^{(2)}E_T^{(1)}E_T^{(2)}\left(\frac{1-\nu_T^{(1)}-\nu_{TL}^{(1)}}{E_T^{(1)}} - \frac{1}{3K^{s(1)}}\right)}{L}P_g \tag{28}$$

Inserting eqns (26)–(28) into eqn (24) and eliminating the volumetric strain of the pore space from the resulting equation and eqn (25), we obtain pore pressure P_g.

$$P_g = \frac{1-(2\beta_T^{(1)}+\beta_L^{(1)})T - \frac{2(\beta_T^{(2)}-\beta_T^{(1)})T}{\Phi^{(1)}}\,\frac{\eta^{(2)}E_T^{(1)}E_T^{(2)}\left(\frac{1-\nu_T^{(1)}-\nu_{TL}^{(1)}}{E_T^{(1)}} - \frac{1}{3K^{s(1)}}\right)}{L}}{\frac{MV_{\mathrm{p1}}}{m_g RT_a} + \frac{1}{\Phi^{(1)}}N} \tag{29}$$

where

$$N = \left(\frac{2(1-\nu_T^{(1)}-\nu_{TL}^{(1)})}{E_T^{(1)}} + \frac{1-2\nu_{LT}^{(1)}}{E_L^{(1)}} - \frac{1+\Phi^{(1)}}{K^{s(1)}}\right) - \frac{2\eta^{(2)}E_T^{(1)}E_T^{(2)}\left(\frac{1-\nu_T^{(1)}-\nu_{TL}^{(1)}}{E_T^{(1)}} - \frac{1}{3K^{s(1)}}\right)^2}{L} \tag{30}$$

The numerator in eqn (29) reduces to one if we set the thermal expansion terms equal to zero. Since pore pressure is positive even if thermal expansion is ignored, the denominator in eqn (29) is positive. The compliance of a porous material is larger than the corresponding compliance of the solid material, we have

$$\frac{1-\nu_T^{(1)}-\nu_{TL}^{(1)}}{E_T^{(1)}} - \frac{1}{3K^{s(1)}} > 0 \tag{31}$$

Since the denominator in eqn (29) is positive, the coefficient for $\beta_T^{(2)}T$ in eqn (29) is negative. When carbon fibers are subjected to shrinkage, $\beta_T^{(2)}T$ is negative. Therefore, the carbon shrinkage in the transverse direction causes higher pore pressure.

Combining eqns (4) and (26), we have

$$\sigma_{33}^{s(1)} = \frac{\Phi^{(1)}}{1-\Phi^{(1)}}P_g. \tag{32}$$

From eqn (32), higher pore pressure leads to higher solid resin stress in the direction perpendicular to the layers. Therefore, given the same ultimate strength of the solid resin, material properties (other than thermal expansion coefficients), delamination occurs more in the composite with carbon shrinkage than without it.

Using the expression of effective constant eqns (16a) and (17a) and the overall constitutive equation in the form of eqn (14a), the overall normal strain in the in-plane direction is given by

$$\varepsilon_{11} = \frac{((1-\nu_T^{(1)})E_T^{(2)}\eta^{(2)}\beta_T^{(2)} + \eta^{(1)}(1-\nu_T^{(2)})\beta_T^{(1)}E_T^{(1)})T}{L} + \frac{E_T^{(1)}\eta^{(1)}(1-\nu_T^{(2)})\left(\dfrac{1-\nu_T^{(1)}-\nu_{TL}^{(1)}}{E_T^{(1)}} - \dfrac{1}{3K^{s(1)}}\right)}{L}P_g \quad (33)$$

Similarly, using eqns (16a), (17b) and the overall constitutive equation in the form of eqn (14c), the overall normal strain in the out-of-plane direction is given by eqn (34).

$$\begin{aligned}\varepsilon_{33} = &-\frac{2\eta^{(1)}\eta^{(2)}}{L}\left(\frac{1-\nu_T^{(1)}-\nu_{TL}^{(1)}}{E_T^{(1)}} - \frac{1}{3K^{s(1)}}\right)(-\nu_{TL}^{(1)}E_T^{(2)} + \nu_{TL}^{(2)}E_T^{(1)})P_g \\ &+\eta^{(1)}\left(\frac{1-2\nu_{LT}^{(1)}}{E_L^{(1)}} - \frac{1}{3K^{s(1)}}\right)P_g + \frac{2\eta^{(1)}\eta^{(2)}(\nu_{TL}^{(2)}E_T^{(1)} - \nu_{TL}^{(1)}E_T^{(2)})}{L} \\ &\times(\beta_T^{(2)}-\beta_T^{(1)})T + (\eta^{(2)}\beta_L^{(2)} + \eta^{(1)}\beta_L^{(1)})T \quad (34)\end{aligned}$$

4. CONCLUSIONS

Explicit expressions of the effective poroelastic parameters for layered porous materials are obtained. Given the material properties of the constituent layers, the effect of pore pressure on overall deformation of the composite can be described by these effective poroelastic parameters. Explicit expressions of the constituent stress components are often not included in the analysis done by others. Given the effective material properties of the composite, macroscopic stress field in boundary value problems can be calculated either analytically or numerically. Using the obtained macroscopic stress field, eqns (21) make it possible for us to evaluate the internal stress components of each constituent layer. The pointwise stress distribution, which varies throughout the thickness of each layer, cannot be obtained by this analysis. However, the internal stress in each layer averaged over the thickness may be sufficient for many practical applications.

This work is motivated by failure analysis of carbon–phenolic composites used as heat liners to protect metal in an extreme thermomechanical environment. The micro-structure of the composite can be idealized as a layered structure of woven carbon fiber and porous matrix.

Carbon–phenolic composites experience delamination known as "ply lift failure". This failure is attributed to high pore pressure caused by gas generated due to phase change. The difference in material properties of fibers in woven carbon fiber layers influences the overall material behavior of the composite and also failure behaviors. The developed model explains why carbon fiber with shrinkage is more likely to cause the ply lift failure than that without it.

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